FLOW METER DISCHARGE COEFFICIENT ESTIMATION

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Abstract:

Calculation of energy losses is critical when taking measurements from a system such as a flowmeter. Knowing the properties of a working fluid in a pipe such as flow rates and pressure is critical to nearly every application of internal flow in order to make the calculations necessary for appropriately sizing a pump or turbine, determining required dimensions of the pipe and fittings, or to avoid hazards such as cavitation. This experiment used a FNE18 Edibon Flow Meter Module in order to calculate the energy losses due to flow through Venturi, orifice and rotameter flowmeters, and estimate the coefficient of discharge for the Venturi and orifice meters. The energy losses were smallest for the Venturi meter ranging from 0.0 to 1.3×10^{-4} kJ/kg and largest for the rotameter at a nearly constant 2.3×10^{-3} kJ/kg. A comparison was made between the experimental and theoretical discharge coefficient calculations. In both cases, the experimental calculation was slightly higher than the theoretical due to experimental error. The resulting values were 0.9899 for the Venturi meter (error of only 1.01 percent) and 0.7113 for the orifice meter (error of 13.5 percent). These values can be used in practice to account for irreversible losses of the systems.

Introduction:

Despite technological advancements, an ideal flowmeter does not exist because some pressure losses due to friction are unavoidable. For this reason, in order to obtain accurate calculations of flow properties by using a flowmeter, it is necessary to determine the discharge coefficient of the equipment being used. The discharge coefficient is defined as the ratio of the actual flow to the flow that would pass through an ideal orifice, convergent nozzle, or Venturi meter (Bragg, 1960). In other words, it is the ratio of the mass flow rate at the discharge end of the obstructive flowmeter to that of an ideal system with the same fluid, under the same conditions. The discharge coefficient is the parameter used to determine the irrecoverable losses across a certain flowmeter or piece of equipment. Keeping track of these losses is important in calculating the efficiency of a system and helps maintain accuracy when taking measurements from various types of flowmeters.

An obstructive flowmeter is used to measure the flow rate through a pipe by "constricting the flow and measuring the decrease in pressure due to the increase in velocity at the constriction site" (Cengel & Cimbala, 2014). A manometer is used to measure the pressure difference before and after the obstruction. The discharge coefficient is a value that represents both the friction loss mentioned previously and the loss due to the vena contracta phenomenon, which is when the fluid stream continues to contract past the point of obstruction (Cengel & Cimbala, 2014). This parameter can be calculated both theoretically and experimentally.
This experiment uses three types of flowmeters. The orifice meter is the simplest and consists of a vertical plate in the pipe with a hole in the middle. Being the simplest, this design also produces substantial losses (frictional, head, pressure, etc.) due to a significant amount of swirl caused by the abrupt change in flow area. Figure 1 shows the typical orifice flow pattern where $P_1 - P_2$ indicates the initial pressure drop and $P_1 - P_3$ gives permanent pressure loss.

![Figure 1: Typical orifice flow pattern using a manometer to measure pressure (“Fundamentals”, 2010)](image)

The second is a Venturi meter, which is the most expensive and most accurate obstruction flowmeter. Its "gradual contraction and expansion prevent flow separation and swirling, and it suffers only frictional losses on the inner wall surfaces" (Cengel & Cimbala, 2014). They are especially useful in applications that cannot allow large pressure drops. The flow pattern of this type of meter is modeled in Figure 2 where it can be seen that the permanent pressure losses are much less than that of the orifice meter in Figure 1.

![Figure 2: Typical Venturi flow pattern using a manometer to measure pressure (“Basic Concepts Related to Flowing Water and Measurement”)](image)

The final flowmeter used in this experiment is a variable-area flowmeter called a rotameter. It consists of a tapered transparent tube and a free moving float inside. The working fluid flows through the tube to a point where the weight and buoyancy forces on the float balance out, leaving
the drag force as the only variable since it increases with fluid velocity. Flow rate in the pipe can be determined by comparing the height of the float in the tube to a scale on the side. The accuracy of a rotameter is generally ±5 percent (Cengel & Cimbala, 2014), making it unsuitable for applications that require a precise measurement. It also leaves room for human error since the measurements depend on visual observations of the location of the float. Figure 3 is a model of the flow pattern of a rotameter.

Figure 3: Typical flow model of a rotameter ("Types of Gas Mass Flow Meters", 2005)

It is important to remember that despite the fact that the discharge coefficient can be estimated experimentally and theoretically, its exact value for any flowmeter depends on the particular design of the system. Therefore, data from the manufacturer should be consulted when it is available.

Objectives:

The goal of this experiment is to use the FNE18 Flow Meter Module from Edibon to estimate the discharge coefficients of the orifice and Venturi meters and quantify energy losses of the orifice, Venturi, and rotameter flowmeters. Careful observations and records as well as understanding of the measurement process are necessary to reach this objective.
Methods:

This experiment uses a FNE18 Flow Meter Module from Edibon to perform observations and calculations. This system consists of Venturi, orifice and rotameter flowmeters, connected to a bank of manometers at critical points, and a timer and dump valve system used to measure and control volumetric flow rate through the pipe. The two adjustable valves were initially set to a flow rate of $8 \times 10^{-5}$ m$^3$/s for 93 seconds and gradually increased over seven trials to $3.8 \times 10^{-4}$ m$^3$/s. The flow rates were calculated by recording the initial and final volume of the tank over a set time. For each flow rate, measurements were taken from the eight manometer taps. Taps 1, 2 and 3 corresponded to the inlet, neck and outlet of the Venturi meter, taps 4 and 5 corresponded to the inlet and outlet of the rotameter, and taps 6, 7 and 8 corresponded to the inlet, constriction site, and outlet of the orifice meter. Once observations were made for each trial, the information was mathematically and graphically evaluated to find the pressure at each point, energy losses for the systems, and the square root of the pressure drops across the flow meters. Plots of the energy losses vs. volumetric flow rates were made from the calculated data. A plot was also made of the volumetric flow rates vs. the square root of the pressure drop across the flow meters. The slope from each plot was used to estimate the discharge coefficient for the Venturi and orifice meters (Equation 1).

$$Slope = C_d A_2 \sqrt{\frac{2}{\rho \left(1 + \frac{A_2}{A_1} \right)}}$$  \hspace{1cm} \text{Equation 1}$$

Where: $Slope = \text{The slope of the trendline from the plot of flow rate vs. the square root of pressure drop}$

$C_d =$ discharge coefficient

$A_1 =$ outer cross-sectional area

$A_2 =$ inner cross-sectional area

$\rho =$ working fluid density

The discharge coefficient can also be determined theoretically for orifice flowmeters using Equation 2. Venturi meters are estimated to have a theoretical discharge coefficient of 0.98 in the absence of specific data, because $C_d$ for these flowmeters are generally very high for most flows (Cengel & Cimbala, 2014). The comparison between the experimental and theoretical coefficients offers an estimation of experimental error and other minor losses.
\[ C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{Re^{0.75}} \]  \hspace{1cm} \text{Equation 2 (Cengel & Cimbala, 2014)}

Where: 
- \( C_d \) = discharge coefficient
- \( \beta = d/D \); ratio of inner diameter to outer diameter
- \( Re = \frac{\rho V_1 D}{\mu}; \) Reynolds’ Number

Results and Discussion:

From the head data collected in this experiment, the energy losses, volumetric flow rates and pressure drops for the Venturi, orifice and rotameter flowmeters are calculated. Figure 4 is a plot of the energy losses vs. the volumetric flow rates for all three flowmeters. The energy losses were determined using the energy equation (Equation 3) where the change in pressure was taken between the inlet and discharge points of the systems where the cross-sectional areas are equal. For the flowmeters used, the energy equation can be reduced to Equation 4 since \( v_1 \approx v_2, z_1 = z_2 \) and \( W_{\text{pump}} \) and \( W_{\text{turbine}} \) equal zero.

\[
\frac{P_1}{\rho} + \alpha \frac{v_1^2}{2} + g z_1 + W_{\text{pump}} = \frac{P_2}{\rho} + \frac{v_2^2}{2} + g z_2 + W_{\text{turbine}} + E_{\text{mech, loss}} \hspace{1cm} \text{Equation 3}
\]

\[
E_{\text{mech, loss}} = \frac{P_1}{\rho} - \frac{P_2}{\rho} \hspace{1cm} \text{Equation 4}
\]

Where: 
- \( P = \) pressure
- \( \rho = \) density of working fluid
- \( \alpha = \) kinetic energy correction factor
- \( g = \) gravity
- \( z = \) height
- \( W_{\text{pump}} = \) power of pump
- \( W_{\text{turbine}} = \) power of turbine
- \( E_{\text{mech, loss}} = \) mechanical loss of energy

The graph below confirms that the Venturi meter is the most efficient method of measurement because it produces very low energy losses over a range of flow rates from \( 5\times10^{-5} \) m\(^3\)/s to \( 4\times10^{-4} \) m\(^3\)/s. As seen from the slope of the trendline for these points, its energy losses increase slightly with volumetric flow rate, however, even at the largest flow rate, the losses are nearly negligible. The rotameter produces the greatest energy losses, which was also discussed in the introductory section of this report. The graph shows a nearly consistent energy loss of \( 2.3\times10^{-3} \) kJ/kg for the same range in volumetric flow rates. The orifice falls in between with a more
considerable increase in energy loss with volumetric flow rate, which is visualized in the slope of its trendline. This is due to the fact that as the velocity of the working fluid increases, the area of the vena contracta becomes steadily smaller than that of the constricted cross-sectional area. As the ratio between these two areas increases, more energy loss is experienced by the system.

![Energy Loss vs. Flow Rate Graph]

**Figure 4: Plot of the energy loss (kJ/kg) vs. the volumetric flow rate (m³/s) for the Venturi, orifice and rotamer flow meters.**

The graph in Figure 5 is a representation of the relationship between the volumetric flow rates and the square roots of the pressure drops across the flowmeters. Trendlines with an intercept of zero are set for the data of the Venturi and orifice meters in order to determine the slope of the observed data, which is used in the calculation of experimental discharge coefficients. As seen in the graph, the pressure drops for the Venturi and orifice meters increase linearly with an increase in flow rate of working fluid. This is due to the fact that pressure drops are sensitive to increases in the velocity of the fluid through these two instruments. The slope of the Venturi plot is steeper because its design prevents large pressure drops, meaning that even at the highest flow rates for this experiment, the difference in pressure between the inlet and outlet does not exceed 25Pa. The more gradual slope of the orifice meter shows that this system is more sensitive to changes in the velocity of the fluid because its pressure drop increases at a greater rate than that of the Venturi. Again, this is due to the fact that the simple design produces more swirling of the fluid, which results in a larger permanent pressure loss. The vertical plot of the rotameter suggests that the pressure drop across this system remains nearly constant at around 47Pa regardless of change.
in flow rate. This may be because even at low flow rates, the pressure drop is large due to the loss of pressure as the fluid flows around the float, causing swirl and friction loss. Therefore, the increase in pressure loss due to changes in fluid velocity is negligible.

![Graph](image)

**Figure 5: Plot of the flow rates (m³/s) vs. the square root of pressure drops (Pa)**

Evaluation of Equation 1 using the slopes of the Venturi and orifice trendlines from the previous figure and the areas given in Table 1 made it possible to determine the discharge coefficient for these two systems.

**Table 1: Measurements of the inner and outer cross-sectional areas of the Venturi and orifice meters**

<table>
<thead>
<tr>
<th></th>
<th>ΔP</th>
<th>A₁ (m²)</th>
<th>A₂ (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Venturi</td>
<td>P₁ – P₂</td>
<td>8.04×10⁻⁴</td>
<td>3.14×10⁻⁴</td>
</tr>
<tr>
<td>Orifice</td>
<td>P₆ – P₇</td>
<td>9.62×10⁻⁴</td>
<td>2.83×10⁻⁴</td>
</tr>
</tbody>
</table>

Solving for the discharge coefficient for the Venturi meter produced a value of 0.9899, which is an error of only 1.01 percent from the theoretical value of 0.98 (Cengel & Cimbala, 2014). The discharge coefficient of the orifice meter had a value of 0.7114 when evaluated experimentally, which is reasonably close to the theoretical value of 0.627 with an experimental error of 13.5
percent. This can be justified by such factors as human error, friction error, or not turning the valves to the same degree. The values calculated for the discharge coefficient of the orifice also agree with those found in an evaluation of orifice plates in *Applied Engineering and Agriculture Vol. 5*, which states that “the discharge coefficient $C_d$ has values of 0.620, 0.638 and 0.675 for orifice diameters of 38 mm (1.5 in), 64 mm (2.5 in) and 89 mm (3.5 in), respectively, in a 152 mm (6 in) diameter riser” (Hue, James & McEnroe, 1989). Although the diameters for these orifices are greater than the one used in this experiment, this excerpt offers a reasonable comparison of results.

The higher coefficient of discharge corresponds with the more efficient Venturi system because the irrecoverable losses are much smaller. The orifice, experiences greater permanent losses, which must be taken into account when selecting what type of flowmeter is appropriate for a given application.

**Conclusion:**

The energy losses and coefficients of discharge were calculated for the flowmeters in this experiment from the observations made from the manometers and the dump valve system of the Edibon Flow Meter Module. The mathematical calculations and graphical analysis of the recorded data determined that the Venturi meter was the most efficient flowmeter because it has the smallest irrecoverable energy losses ranging from 0.0 to $1.3 \times 10^{-4}$ kJ/kg over seven trials at different flow rates. The orifice meter was found to have energy losses that increased with fluid velocity due to fluid swirling after the orifice plate. Losses range from $5.9 \times 10^{-5}$ to $1.1 \times 10^{-3}$ kJ/kg. The rotameter was deemed the least efficient with consistent losses of around $2.3 \times 10^{-3}$ kJ/kg for the seven different flow rates. The coefficients of discharge for the Venturi and orifice meters were determined from the slope of the trendlines of the flow rate vs. square root of pressure drop graph and the cross-sectional areas of the pipe and neck. The experimentally calculated $C_d$ for the Venturi meter was almost an exact match to the theoretically estimated value of 0.98 with an error of only 1.01 percent, which can be justified by factors such as human error and friction. The experimentally calculated $C_d$ for the orifice meter was a value of 0.711, which had acceptably small error of 13.5 percent from the theoretical coefficient.
References:


